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Dilaton Condensation in Cubic Open String Field Theory

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Abstract

We construct an exact and finite classical solution in cubic open string field theory. The classical solution has a well-defined Fock space expression. Through the redefinition of a string field, the string field theory expanded around the classical solution can be transformed into the theory with a different coupling constant, where other backgrounds are unchanged. Therefore, the classical solution represents the condensation of the dilaton in cubic open string field theory.

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1 Introduction

Cubic open string field theory (CSFT) [1] has a classical solution corresponding to the condensation of tachyons living on D-branes [2, 3]. If an analytic solution of the tachyon condensation is found, we may construct a new string field theory without D-branes. Though the analytic solution is lacking, it has been proposed that vacuum string field theory (VSFT) describes the tachyon vacuum without D-branes, and it has been under intense investigations [4]. However, we have not yet arrived at the complete formulation of VSFT because it needs a regularization which is still unknown [5]. At present, the exact solution in CSFT is necessary for the construction of a string field theory around the tachyon vacuum.

While it is difficult to find an analytic solution of the tachyon condensation, there is an exact solution representing the condensation of gauge fields, namely Wilson lines background [6]. The solution is given by the multiplication of some operators and the identity string field. Then, the equation of motion can be reduced to the algebraic equation of the operators. A marginal tachyon lump solution is also provided in a similar way.

In this paper, we present a new exact and finite classical solution in CSFT with a well-defined Fock space expression. We show that the solution can be constructed by the identity string field and some operators, as in the case of the Wilson lines solution. Through a homogeneous redefinition of a string field, we find that the string field theory around the classical solution describes interacting strings in the almost same background as the original theory, where only the string coupling constant has a different value. Therefore, the new solution represents the condensation of dilaton in CSFT, which is an excitation mode of closed strings.

The dilaton condensation in string field theory was first investigated in the light-cone closed string field theory [7]. Subsequently, it was considered in the HIKKO type string field theory [8, 9, 10] and the non-polynomial closed string field theory [11]. In the case of CSFT, it was not clear how to treat the dilaton condensation, because it is difficult to identify the dilaton as a component of the string field in contrast to other theories using a closed string field. However, it is believed that a string field in CSFT contains the dilaton and other closed string modes, since one loop amplitudes have closed string poles in CSFT [12] and unitarity requires that the string field includes closed strings as external states. To this long-standing puzzle, our classical solution answers that the string field contains at least a zero-momentum dilaton state.

This paper is organized as follows. In sec. 2, we construct a classical solution in CSFT. We find that the solution has a well-defined Fock space expression. In sec. 3, considering a redefinition of the string field, we show that the classical solution represents the dilaton condensation. In sec. 4, we give summary and discussions. We present some detail calculations in the appendix.

2 Classical Solution of String Field Theory

The BRS current is defined by

$$J_{\rm B}(w) = c\left(T_X + \frac{1}{2}T_{\rm gh}\right)(w) + \frac{3}{2}\partial^2 c(w), \tag{2.1}$$

where $c(w)$ is a ghost field and $T_X(w)$ and $T_{\rm gh}(w)$ denote the energy momentum tensors of string coordinates and reparametrization ghosts, respectively [13]. The operator product expansions (OPEs) of the BRS current and the ghost field are given by

$$\begin{aligned} J_{\rm B}(w)J_{\rm B}(w') &= \frac{-(d-18)/2}{(w-w')^3}c\partial c(w') + \frac{-(d-18)/4}{(w-w')^2}c\partial^2 c(w') - \frac{(d-26)/12}{w-w'}c\partial^3 c(w') + \cdots \\ &= \frac{-4}{(w-w')^3}c\partial c(w') + \frac{-2}{(w-w')^2}c\partial^2 c(w') + \cdots, \\ J_{\rm B}(w)c(w') &= \frac{1}{w-w'}c\partial c(w') + \cdots, \end{aligned} \tag{2.2}$$

where $d = 26$ is the matter central charge of the conformal field theory. We can expand the BRS current and the ghost field using oscillation modes,

$$\begin{aligned} J_{\rm B}(w) &= \sum_{n=-\infty}^{\infty} Q_n w^{-n-1}, \\ c(w) &= \sum_{n=-\infty}^{\infty} c_n w^{-n+1}. \end{aligned} \tag{2.3}$$

Since $\{Q_{\rm B},\, c(w)\} = c\partial c(w)$, the OPEs of Eq. (2.2) can be rewritten in the form of anti-commutation relations of these oscillators,

$$\{Q_m,\, Q_n\} = 2mn\{Q_{\rm B},\, c_{m+n}\}, \quad \{Q_m,\, c_n\} = \{Q_{\rm B},\, c_{m+n}\}. \tag{2.4}$$

We now define the following operators,

$$\begin{aligned} Q_{\rm L}(f) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} d\sigma f(\sigma) J_{\rm B}(\sigma), \quad Q_{\rm R}(f) = \frac{1}{\pi}\int_{\frac{\pi}{2}}^{\pi} d\sigma f(\sigma) J_{\rm B}(\sigma), \\ C_{\rm L}(f) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} d\sigma f(\sigma) C(\sigma), \quad C_{\rm R}(f) = \frac{1}{\pi}\int_{\frac{\pi}{2}}^{\pi} d\sigma f(\sigma) C(\sigma), \end{aligned} \tag{2.5}$$

where $f(\sigma)$ is an arbitrary function with the Neumann boundary condition, and the BRS current $J_{\rm B}(\sigma)$ and the ghost field $C(\sigma)$ are given by

$$\begin{aligned} J_{\rm B}(\sigma) &= \sum_{n=-\infty}^{\infty} Q_n \cos(n\sigma), \\ C(\sigma) &= \sum_{n=-\infty}^{\infty} c_n \cos(n\sigma). \end{aligned} \tag{2.6}$$

Supposed that a function $f(\sigma)$ satisfies $f(\pi-\sigma) = f(\sigma)$, we find from the connection conditions on the identity string field

$$Q_{\rm L}(f)I = -Q_{\rm R}(f)I, \quad C_{\rm L}(f)I = -C_{\rm R}(f)I, \tag{2.7}$$

where, as an additional restriction, $f(\sigma \to \pi/2)$ must be zero so that the operator $C_{\rm L(R)}$ may be well-defined on the identity string field [6]. From the connection conditions on the three string vertex, the operators of $Q_{\rm L(R)}$ and $C_{\rm R(L)}$ satisfy

$$\begin{aligned} (Q_{\rm R}(f)A) * B &= -(-1)^{|A|} A * (Q_{\rm L}(f)B)\,, \\ (C_{\rm R}(f)A) * B &= -(-1)^{|A|} A * (C_{\rm L}(f)B)\,, \end{aligned} \tag{2.8}$$

where A and B are arbitrary string fields, and $|A|$ is 0 if A is Grassmann even and 1 if it is odd.

Let us consider the commutation relations of the operators $Q_{\rm L(R)}$ and $C_{\rm L(R)}$. From Eq. (2.4), we can derive

$$\{Q(f),\, C_{\rm L}(g)\} = \{Q_{\rm B},\, C_{\rm L}(fg)\}, \tag{2.9}$$

where the operator $Q(f)$ is defined by the summation of $Q_{\rm L}$ and $Q_{\rm R}$,

$$Q(f) = Q_{\rm L}(f) + Q_{\rm R}(f) = \int_0^\pi \frac{d\sigma}{\pi} f(\sigma) J_{\rm B}(\sigma). \tag{2.10}$$

Here, we take $h(\sigma) = 1 + \cos(2\sigma)$ as the function in $Q_{\rm L}$. This function is one of the simple choices and we can not choose $h(\sigma) = 1$ as discussed later soon. Using oscillators, the operator $Q_{\rm L}(h)$ is written by

$$Q_{\rm L}(h) = \frac{1}{2}Q_0 + \frac{1}{4}\left(Q_2 + Q_{-2}\right) - \frac{4}{\pi}\sum_{n\neq 0,\pm 2} \frac{1}{n(n^2-4)} Q_n \sin\left(\frac{n\pi}{2}\right). \tag{2.11}$$

From Eq. (2.4), we can calculate the commutation relations of $Q_{\rm L}$s,

$$\begin{aligned}
\{Q_{\rm L}(h),\, Q_{\rm L}(h)\} &= -\{Q_{\rm B},\, c_0 - \frac{1}{2}(c_4 + c_{-4})\} \\
&\quad + \frac{64}{\pi} \sum_{n\neq 0,\pm 4} \frac{1}{n(n^2-16)} \{Q_{\rm B},\, c_n\} \sin\left(\frac{n\pi}{2}\right) \\
&\quad + \frac{32}{\pi^2} \sum_m a_m \{Q_{\rm B},\, c_m\},
\end{aligned} \tag{2.12}$$

where a_m are given by

$$\begin{aligned}
a_m &= \sum_{n\neq 0,\pm 2, m, m\pm 2} \frac{1}{(n^2-4)((n-m)^2-4)} \sin\left(\frac{n\pi}{2}\right) \sin\left(\frac{(n-m)\pi}{2}\right) \\
&= \frac{1}{16} \sum_{n\neq 0, m, m\pm 4} \left[\frac{2}{n(n-m)} - \frac{1}{n(n-m+4)} - \frac{1}{n(n-m-4)}\right] \\
&\quad \times \sin\left(\frac{n\pi}{2}\right) \sin\left(\frac{(n-m)\pi}{2}\right).
\end{aligned} \tag{2.13}$$

We can sum up the infinite series a_m and obtain the commutation relation

$$\{Q_{\rm L}(h),\, Q_{\rm L}(h)\} = -4\{Q_{\rm B},\, C_{\rm L}(1-\cos(4\sigma))\}, \tag{2.14}$$

where use has been made of the formula [6]

$$\sum_{n\neq 0, m} \frac{1}{n(n-m)} \sin\left(\frac{n\pi}{2}\right) \sin\left(\frac{(n-m)\pi}{2}\right) = \frac{\pi^2}{4}\delta_{m,0}. \tag{2.15}$$

It can be seen that if we choose $f(\sigma) = 1$ or $\cos(2\sigma)$, or other function which satisfies $f(\sigma \to \pi/2) \neq 0$, the infinite series like a_m appeared in the commutator does not convergent. The divergence is due to the midpoint singularity in the commutation relation. Therefore, the function $f(\sigma)$ in $Q_{\rm L}$ should be constrained as $f(\sigma \to \pi/2) = 0$ in order to make the anti-commutator of $Q_{\rm L}$s be well-defined without the midpoint singularity.

We should comment on a pregeometrical formulation of string field theory [14], in which a classical solution $Q_{\rm L}(1)I$ satisfies the equation of motion that is $Q_{\rm L}(1)I * Q_{\rm L}(1)I = Q_{\rm L}(1)^2 I = 0$. However, this is somewhat formal discussion. We find from the oscillator calculation that $Q_{\rm L}(1)^2 I$ suffers from the divergence, as discussed above. If there is a better language of the string field theory than the oscillators expansion, we can regulate the divergence and define the pregeometrical formulation. Otherwise, it seems failure to formulate the purely cubic theory.

Now, we can show that a classical solution in the cubic open string field theory is given by

$$\Psi_0(a) = Q_{\rm L}(ah)I + C_{\rm L}(\eta_a)I, \tag{2.16}$$

where $h(\sigma) = 1 + \cos(2\sigma)$ and $\eta_a(\sigma)$ is

$$\eta_a(\sigma) = 4a^2 \frac{\sin^2(2\sigma)}{1 + 2a\cos^2\sigma}, \tag{2.17}$$

and a is a real parameter which is larger than $-1/2$. Indeed, from Eqs. (2.7)–(2.9) and (2.14), it follows that

$$\begin{aligned}
Q_{\rm B}\Psi_0 &= \{Q_{\rm B},\, C_{\rm L}(\eta_a)\}I,\\
\Psi_0 * \Psi_0 &= Q_{\rm L}(ah)I * Q_{\rm L}(ah)I\\
&\quad + Q_{\rm L}(ah)I * C_{\rm L}(\eta_a)I + C_{\rm L}(\eta_a)I * Q_{\rm L}(ah)I\\
&= \frac{1}{2}\{Q_{\rm L}(ah),\, Q_{\rm L}(ah)\}I + \{Q(ah),\, C_{\rm L}(\eta_a)\}I\\
&= -2a^2\{Q_{\rm B},\, C_{\rm L}(1-\cos(4\sigma))\}I + \{Q_{\rm B},\, C_{\rm L}(ah\,\eta_a)\}I.
\end{aligned} \tag{2.18}$$

Then, Ψ_0 obeys the equation of motion

$$\begin{aligned}
Q_{\rm B}\Psi_0 + \Psi_0 * \Psi_0 &= \{Q_{\rm B},\, C_{\rm L}(\eta_a)\}I - 2a^2\{Q_{\rm B},\, C_{\rm L}(1-\cos(4\sigma))\}I + \{Q_{\rm B},\, C_{\rm L}(ah\,\eta_a))\}I\\
&= \{Q_{\rm B},\, C_{\rm L}(\eta_a - 2a^2(1-\cos(4\sigma)) + ah\,\eta_a)\}I\\
&= 0.
\end{aligned} \tag{2.19}$$

It should be noted that the classical solution is background independent, since, in order to prove Eq. (2.16) to be a classical solution, we have only to use the relations of Eqs. (2.7)–(2.9) and (2.14) which do not refer to any specific background.

We consider the Fock space expression of the classical solution Eq. (2.16). From the conservation law of the BRS current on the identity string field, it follows that

$$\left(Q_n + (-1)^n Q_{-n}\right)|I\rangle = 0, \tag{2.20}$$

and then we find

$$J_{\rm B}(\sigma)\,|I\rangle = \sum_{n=0}^{\infty} Q_{-2n-1}\cos[(2n+1)\sigma]\,|I\rangle\,, \tag{2.21}$$

which are similar equations to α_n's, where α_n is the oscillator of the holomorphic dimension one current ∂X as same as Q_n in J_B. The oscillator expression of the identity string field is given by [6, 15, 16]

$$
\begin{aligned}
&|I\rangle = e^{E}\,|0\rangle\,,\\
&E = -\sum_{n\geq 1}\frac{(-)^n}{2n}\alpha_{-n}\cdot\alpha_{-n} + \sum_{n\geq 2}(-)^n c_{-n}b_{-n}\\
&\quad -2c_0\sum_{n\geq 1}(-)^n b_{-2n} - (c_1 - c_{-1})\sum_{n\geq 1}(-)^n b_{-2n-1},
\end{aligned}
\tag{2.22}
$$

where we omit the zero mode delta function. Then, the ghost field on the identity string field is written by [6]

$$
C(\sigma)\,|I\rangle = \Big[c_1\,\frac{1}{2\cos\sigma} + c_{-1}\,\frac{1+2\cos(2\sigma)}{\cos\sigma} + 2\sum_{n=1}^{\infty} c_{-2n-1}\,\cos[(2n+1)\sigma]\Big]\,|I\rangle\,. \tag{2.23}
$$

Using Eqs. (2.21), (2.22) and (2.23) we can expand the first and second terms of Eq. (2.16) up to first few levels,

$$
\begin{aligned}
Q(ah)\,|I\rangle &= a\Big[\frac{8}{3\pi}Q_{-1} + \frac{8}{15\pi}Q_{-3} + \frac{4}{3\pi}Q_{-1}(\alpha_{-1}\cdot\alpha_{-1})\\
&\quad +\frac{16}{3\pi}Q_{-1}c_0b_{-2} + \frac{8}{3\pi}Q_{-1}c_1b_{-3}\Big]\,|0\rangle + \cdots,\\
C_{\mathrm{L}}(\eta_a)\,|I\rangle &= \frac{4a}{\pi}\Big[J_1(a)\,c_1 + J_2(a)\,c_{-1}\\
&\quad +\frac{1}{2}J_1(a)\,c_1(\alpha_{-1}\cdot\alpha_{-1}) + 2J_1(a)\,c_1c_0b_{-2}\Big]\,|0\rangle + \cdots,
\end{aligned}
\tag{2.24}
$$

where $J_1(a)$ and $J_2(a)$ are defined by

$$
\begin{aligned}
J_1(a) &= \int_0^{\frac{\pi}{2}} d\sigma\frac{a\sin^2(2\sigma)}{1+2a\cos^2\sigma}\frac{1}{2\cos\sigma},\\
J_2(a) &= \int_0^{\frac{\pi}{2}} d\sigma\frac{a\sin^2(2\sigma)}{1+2a\cos^2\sigma}\frac{1+2\cos(2\sigma)}{2\cos\sigma}.
\end{aligned}
\tag{2.25}
$$

In order to evaluate these integrals, we can use the following formula, which is derived in Appendix A,

$$
\begin{aligned}
\frac{a\sin\sigma\cos\sigma}{1+2a\cos^2\sigma} &= -\sum_{n=1}^{\infty}(-1)^n Z(a)^n\sin(2n\sigma),\\
Z(a) &= \frac{1+a-\sqrt{1+2a}}{a}.
\end{aligned}
\tag{2.26}
$$

The results of the calculations are (the details are given in Appendix B)

$$J_1(a) = \begin{cases} -1 + \dfrac{1}{2}\left(\sqrt{Z(a)} + \dfrac{1}{\sqrt{Z(a)}}\right) \log \dfrac{1+\sqrt{Z(a)}}{1-\sqrt{Z(a)}} & (a \geq 0) \\ -1 - \left(\sqrt{-Z(a)} - \dfrac{1}{\sqrt{-Z(a)}}\right) \arctan\sqrt{-Z(a)} & \left(0 > a > -\frac{1}{2}\right), \end{cases} \tag{2.27}$$

$$J_2(a) = \begin{cases} \dfrac{1}{3} + Z(a) + \dfrac{1}{Z(a)} - \dfrac{1}{2}\left(Z(a)\sqrt{Z(a)} + \dfrac{1}{Z(a)\sqrt{Z(a)}}\right) \log \dfrac{1+\sqrt{Z(a)}}{1-\sqrt{Z(a)}} & (a \geq 0) \\ \dfrac{1}{3} + Z(a) + \dfrac{1}{Z(a)} + \left(Z(a)\sqrt{-Z(a)} - \dfrac{1}{Z(a)\sqrt{-Z(a)}}\right) \arctan\sqrt{-Z(a)} & \left(0 > a > -\frac{1}{2}\right). \end{cases} \tag{2.28}$$

Thus, the classical solution of Eq. (2.16) has a well-defined Fock space expression.

Let us consider the string field theory around the classical solution. The original action is given by

$$S[\Psi] = \frac{1}{g}\int \left(\Psi * Q_{\rm B}\Psi + \frac{2}{3}\,\Psi * \Psi * \Psi\right), \tag{2.29}$$

where g is the coupling constant. If we expand the string field as

$$\Psi = \Psi_0(a) + \Psi', \tag{2.30}$$

the action becomes

$$S'[\Psi'] = S[\Psi_0(a) + \Psi'] \quad = \quad = \frac{1}{g}\int \left(\Psi' * Q'_{\rm B}\Psi' + \frac{2}{3}\,\Psi' * \Psi' * \Psi'\right), \tag{2.31}$$

where the shifted BRS charge is given by $Q'_{\rm B} = Q_{\rm B} + D_{\Psi_0}$, in which D_{Ψ_0} is defined by $D_{\Psi_0}A = \Psi_0 * A - (-)^{|A|} A * \Psi_0$ for any string field A. From Eq. (2.8), the shifted BRS charge becomes

$$Q'_{\rm B} = Q_{\rm B} + Q(ah) + C(\eta_a), \tag{2.32}$$

where $Q(f)$ is defined by Eq. (2.10) and $C(f)$ is defined by

$$C(f) = C_{\rm L}(f) + C_{\rm R}(f) = \int_0^\pi \frac{d\sigma}{\pi} f(\sigma) C(\sigma). \tag{2.33}$$

Here, the shifted action does not involve the potential hight $S[\Psi_0]$ at the classical solution because it is zero [10]. Actually, since $\Psi_0(a)$ is a classical solution for any a $(> -1/2)$, it follows that

$$\frac{d}{da}S[\Psi_0(a)] = \frac{2}{g}\int (Q_{\rm B}\Psi_0 + \Psi_0 * \Psi_0) * \frac{d\Psi_0}{da} = 0. \tag{2.34}$$

Then, we find that $S[\Psi_0(a)] = S[\Psi_0(a=0)] = 0$.

We expect that a is a marginal parameter because the potential hight is zero for any value of a $(> -1/2)$. If we take the Minkowski space as the background, the solution of Eq. (2.16) is Lorentz invariant. Therefore, the solution is expected to represent the condensation of the dilaton, which is only massless scalar in the Minkowski space. In the next section, we will see that the solution corresponds to the dilaton condensation through the redefinition of the string field.

3 Redefinition of String Field

The ghost number currents are given by

$$J_{\rm gh}(w) = cb(w), \tag{3.35}$$

where $c(w)$ and $b(w)$ are ghost and anti-ghost fields, respectively. The OPEs of the ghost current with the BRS current and with the ghost field are given by

$$\begin{aligned} J_{\rm gh}(w)J_{\rm B}(w') &= \frac{4}{(w-w')^3}c(w') + \frac{2}{(w-w')^2}\partial c(w') + \frac{1}{(w-w')^2}J_{\rm B}(w') + \cdots, && (3.36)\\ J_{\rm gh}(w)c(w') &= \frac{1}{w-w'}c(w') + \cdots. && (3.37) \end{aligned}$$

We introduce three operators for an arbitrary holomorphic function $f(w)$,

$$\begin{aligned} &q(f) = \oint \frac{dw}{2\pi i} f(w) J_{\rm gh}(w),\\ &\tilde{Q}(f) = \oint \frac{dw}{2\pi i} f(w) J_{\rm B}(w), \quad \tilde{C}(f) = \oint \frac{dw}{2\pi i} f(w) c(w). \end{aligned} \tag{3.38}$$

The OPEs of Eqs. (3.36) and (3.37) give the following commutation relations,

$$[q(f),\, \tilde{Q}(g)] = \tilde{Q}(fg) - 2\,\tilde{C}(\partial f \partial g), \tag{3.39}$$

$$[q(f),\, \tilde{C}(g)] = \tilde{C}(fg). \tag{3.40}$$

From the commutation relations of Eqs. (3.39) and (3.40), we find that, through the transformation generated by $q(f)$, the BRS charge becomes

$$\begin{aligned} e^{q(f)}\, Q_{\rm B}\, e^{-q(f)} &= Q_{\rm B} + [q(f),\, Q_{\rm B}] + \frac{1}{2!}[q(f),\, [q(f),\, Q_{\rm B}]] + \cdots \\ &= Q_{\rm B} + \tilde{Q}(f) + \frac{1}{2!}\left\{\tilde{Q}(f^2) - 2\tilde{C}((\partial f)^2)\right\} + \cdots \\ &= \tilde{Q}(e^f) - \tilde{C}\left((\partial f)^2 e^f\right). \end{aligned} \tag{3.41}$$

By the mapping $w = \exp(i\sigma)$, the shifted BRS charge of Eq. (2.32) is written by

$$Q'_{\rm B} = Q_{\rm B} + \tilde{Q}\left(\frac{a}{2}(w+w^{-1})^2\right) + \tilde{C}\left(-\frac{a^2}{w^2}\frac{(w^2-w^{-2})^2}{1+\frac{a}{2}(w+w^{-1})^2}\right). \tag{3.42}$$

Using Eq. (3.41), we can rewrite it as follows,

$$Q'_{\rm B} = \tilde{Q}(e^{\xi_a}) - \tilde{C}\left((\partial\xi_a)^2 e^{\xi_a}\right) = e^{q(\xi_a)} Q_{\rm B} e^{-q(\xi_a)}, \tag{3.43}$$

where ξ_a is defined by

$$\xi_a(w) = \log\left(1 + \frac{a}{2}\left(w + \frac{1}{w}\right)^2\right) = \log\left(1 + 2a\cos^2\sigma\right). \tag{3.44}$$

Thus, the shifted BRS charge can be transformed into the original BRS charge through the transformation generated by $q(\xi_a)$. Therefore, if we redefine the string field Ψ' to $e^{-q(\xi_a)}\Psi'$ in the shifted action, the kinetic term transforms into the original form with the BRS charge $Q_{\rm B}$.

Supposed that a classical solution is found and the shifted BRS charge is given by the BRS current and the ghost field in the following,

$$Q'_{\rm B} = \tilde{Q}(F) + \tilde{C}(G), \tag{3.45}$$

where $F(w)$ and $G(w)$ are holomorphic functions. From the OPEs of Eq. (2.2), it follows that

$$\{Q'_{\rm B},\, Q'_{\rm B}\} = 2\{Q_{\rm B},\, C(\,(\partial F)^2 + FG\,)\}. \tag{3.46}$$

Since the shifted BRS charge must be nilpotent, which is assured by the equation of motion, the functions satisfy

$$(\partial F)^2 + FG = 0. \tag{3.47}$$

If we take $F = \pm e^f$, G is given by

$$G = -(\partial F)^2/F = \mp(\partial f)^2 e^f. \tag{3.48}$$

As a result, from Eq. (3.41), we find that the shifted BRS charge of the form of Eq. (3.45) can be transformed into the original BRS charge Q_{B}, or $-Q_{\mathrm{B}}$, which corresponds to D-branes with a negative tension.

Let us recall the definition of the three string vertex in order to see how the cubic term is transformed by the redefinition of the string field. The three string vertex is defined by [17, 18, 19]

$$\langle V_3(3,2,1)|\,|\phi_1\rangle_1\,|\phi_2\rangle_2\,|\phi_3\rangle_3 = \langle h_1[\phi_1]h_2[\phi_2]h_3[\phi_3]\rangle\,, \tag{3.49}$$

where $\langle\rangle$ on the right hand side represents the correlation function referring to the conformal field theory on a complex plane. Each string state $|\phi\rangle_i$, which is a state in i-string Fock space ($i = 1, 2, 3$), is given in the form

$$|\phi\rangle_i = \phi(0)\,|0\rangle_i\,, \tag{3.50}$$

where $\phi(w_i)$ is an operator defined on a unit disc $|w_i| \le 1$ of i-string. Each $h_i(w_i)$ represents the conformal mapping of w_i-plane into the complex plane, and $h_i[\phi]$ means the conformal transform of $\phi(0)$ by h_i. In the cubic open string field theory, we can give three conformal mappings in the following

$$\begin{aligned} h_1(w_1) &= e^{\frac{2\pi}{3}i}\left(\frac{1+iw_1}{1-iw_1}\right)^{\frac{2}{3}}, \\ h_2(w_2) &= \left(\frac{1+iw_2}{1-iw_2}\right)^{\frac{2}{3}}, \\ h_3(w_3) &= e^{-\frac{2\pi}{3}i}\left(\frac{1+iw_3}{1-iw_3}\right)^{\frac{2}{3}}. \end{aligned} \tag{3.51}$$

These mappings transform three unit half-disks (unit disks) into a unit disk (a whole complex plane) as depicted in Fig. 1. Therefore, the correlation function on the right hand side in Eq. (3.49) is defined on the unit disk (the whole complex z-plane) with the boundary on the unit circle. The midpoint and its anti-holomorphic counterpart of each string correspond to the origin and the infinity in the z-plane, respectively.

We can derive the conservation law of $q(f)$ on the vertex according to Ref. [20]. Supposed that $f(z)$ is an analytic scalar, and holomorphic everywhere except possible

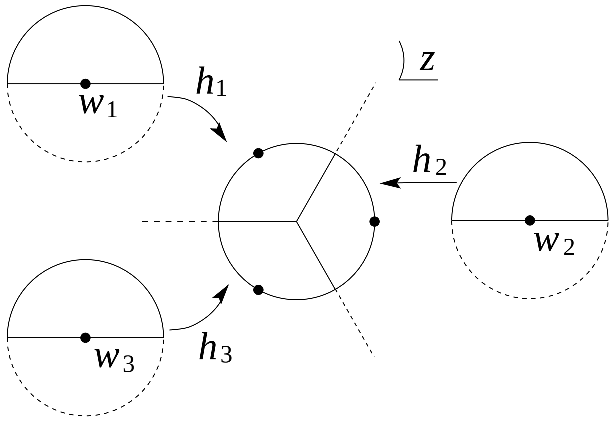

Figure 1: The z-plane representing the three string vertex as a 3-punctured unit disk. The lower unit half disks within the dashed lines on the w_i-disks correspond to the anti-holomorphic parts of the open strings. The conformal mappings of h_i transform the three unit w_i-disks into the whole complex z-plane.

poles at the three punctures in the z-plane. Consider a contour C which encircles the three punctures at the i-strings as depicted in Fig. 2. Since there is no singularity except at the punctures, we find, by the deformation of the contour,

$$\langle V_3| \oint_C \frac{dz}{2\pi i} f(z) J_{\rm gh}(z) = 0. \tag{3.52}$$

We can express the contour integral around the i-string's puncture in terms of the local coordinate w_i. Since the transformation low of the ghost number current $J_{\rm gh}$ is given by

$$\frac{dz}{dw} J_{\rm gh}(z) = J_{\rm gh}(w) + \frac{3}{2} \frac{d^2 z}{dw^2} \left(\frac{dz}{dw}\right)^{-1}, \tag{3.53}$$

we obtain the following identity [20],

$$\begin{aligned} &\langle V_3| \sum_{k=1}^{3} \oint_{C_k} \frac{dw_k}{2\pi i} f(w_k)\, J_{\rm gh}(w_k) = \kappa(f)\, \langle V_3|\,, \\ &\kappa(f) = -\frac{3}{2} \sum_{k=1}^{3} \oint_{C_k} \frac{dw_k}{2\pi i} f(w_k) \frac{d^2 z}{d{w_k}^2} \left(\frac{dz}{dw_k}\right)^{-1}. \end{aligned} \tag{3.54}$$

From Eq. (3.54), it follows that

$$\langle V_3| \prod_{k=1}^{3} e^{q^{(k)}(f)} = e^{\kappa(f)}\, \langle V_3|\,. \tag{3.55}$$

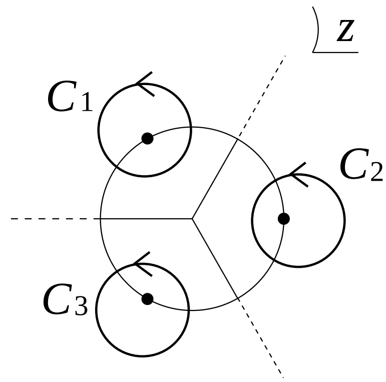

Figure 2: The contour C is given by the summation of the three contours C_i around the punctures.

Now, we consider the redefinition of the string field in the shifted string field theory around the classical solution. If we redefine the string field as

$$\tilde{\Psi} = e^{\kappa(\xi_a)} \left(e^{-q(\xi_a)} \Psi' \right), \tag{3.56}$$

we find, from Eqs. (3.41) and (3.55), that the action of Eq. (2.31) transforms into

$$\begin{aligned} S'[\tilde{\Psi}] &= \frac{1}{g'} \int \left(\tilde{\Psi} * Q_{\mathrm{B}} \tilde{\Psi} + \frac{2}{3} \tilde{\Psi} * \tilde{\Psi} * \tilde{\Psi} \right), & (3.57) \\ g' &= e^{2\kappa(\xi_a)} g. & (3.58) \end{aligned}$$

As a result, through the redefinition of Eq. (3.56), the shifted action becomes the same form of the original action except the coupling constant. Thus, if we expand the string field around the classical solution, the result is the string field theory with the different coupling constant as Eq. (3.58). Therefore, we conclude that the classical solution represents the condensation of the dilaton and the string field in CSFT includes a closed string excitation mode.

Let us consider the moduli space which is swept by the parameter a in the classical solution. In order to evaluate $\kappa(\xi_a)$, we can use the Fourier expansion of ξ_a, which is derived in Appendix A,

$$\begin{aligned} \xi_a(\sigma) &= \log(1 + 2a \cos^2 \sigma) \\ &= -\log \frac{Z(a)}{2a} - 2 \sum_{n=1}^{\infty} \frac{(-1)^n}{n} Z(a)^n \cos(2n\sigma). \end{aligned} \tag{3.59}$$

Using this formula, we find

$$\begin{aligned}\kappa(\xi_a) &= -9\oint\frac{dw}{2\pi i}\,\xi_a(w)\frac{w}{1+w^2} \\ &= -9\sum_{n=1}^{\infty}\frac{1}{n}Z(a)^n \\ &= 9\log\left(1-Z(a)\right) \\ &= -9\log\left(\frac{1+\sqrt{1+2a}}{2}\right). \end{aligned} \tag{3.60}$$

Since $a > -1/2$, $\kappa(\xi_a)$ has the value from $-\infty$ to $9\log 2$ and the redefined coupling constant of Eq. (3.58) takes the value of $g' < 2^{18}g$. Therefore, the classical solution does not cover the full moduli space of the dilaton. However, in order to construct the classical solution generating the coupling constant larger than $2^{18}g$, we have only to use the procedure of the redefinition iteratively. For example, we can obtain the classical solution with the coupling $g' < 2^{36}g$ in terms of the solution and the redefinition of the shifted theory. The resulting solution is given by

$$\Psi_0' = Q_{\mathrm{L}}(a_1 h)I + C_{\mathrm{L}}(\eta_{a_1})I + e^{-\kappa(\xi_{a_1})}e^{q(\xi_{a_1})}\left(Q_{\mathrm{L}}(a_2 h)I + C_{\mathrm{L}}(\eta_{a_2})I\right), \tag{3.61}$$

where a_1 and a_2 are real parameters larger than $-1/2$. When a_1 and a_2 goes to $-1/2$, the solution induces the coupling constant approaching $2^{36}g$. Thus, we can construct the classical solution covering the full moduli space of the dilaton except the point at $g' = \infty$ by the finite iterative procedure.

Finally, let us consider the operator $\exp q(\xi_a)$. Using the Fourier expansion of Eq. (3.59), the operator $q(\xi_a)$ can be expressed by the mode expansion form,

$$\begin{aligned} q(\xi_a) &= -q_0\log\frac{Z(a)}{2a} + \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}(q_{2n}+q_{-2n})Z(a)^n \\ &= -q_0\log\frac{Z(a)}{2a} + q^{(+)}(\xi_a) + q^{(-)}(\xi_a), \end{aligned} \tag{3.62}$$

where $q^{(+)}$ and $q^{(-)}$ denote the positive and negative modes part of q, namely

$$q^{(+)}(\xi_a) = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}q_{2n}Z(a)^n, \quad q^{(-)}(\xi_a) = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}q_{-2n}Z(a)^n. \tag{3.63}$$

From the OPE of the ghost number currents

$$J_{\mathrm{gh}}(w)J_{\mathrm{gh}}(w') = \frac{1}{(w-w')^2} + \cdots, \tag{3.64}$$

the oscillator q_n satisfies $[q_m,\, q_n] = m\delta_{m+n}$. Therefore, we find the commutation relation of $q^{(\pm)}$ as follows,

$$[q^{(+)}(\xi_a),\, q^{(-)}(\xi_a)] = 2\sum_{n=1}^{\infty}\frac{1}{n}Z(a)^{2n} = -2\log(1 - Z(a)^2). \tag{3.65}$$

Using Eq. (3.65), we can rewrite the operator $\exp q(\xi_a)$ by the 'normal ordered' form

$$e^{q(\xi_a)} \;\; = \;\; = \left(1 - Z(a)^2\right)^{-1} \exp\left(-q_0 \log\frac{Z(a)}{2a}\right) e^{q^{(-)}(\xi_a)} e^{q^{(+)}(\xi_a)}. \tag{3.66}$$

This is a well-defined operator since $|Z(a)| < 1$ for $a > -1/2$. Therefore, the redefinition of the string field is well-defined in the Fock space expression.

4 Summary and Discussion

We construct an exact and finite classical solution, around which the expanded string field theory has a different coupling constant from the original theory. Therefore, we conclude that the solution represents the dilaton condensation. The classical solution has a well-defined Fock space expression. Though, so far, we have considered the solution given by the function $h(\sigma) = 1 + \cos 2\sigma$, it is easily generalized to cases of other functions.

In our solution, the operators on the identity string field are defined by the half-string integration of the BRS current and the ghost field multiplied by some functions. On the other hand, there is an old idea that closed string states in CSFT are given by the operator on the identity string field which is the half-string integration of a closed string vertex operator [21]. Though our solution is similar to this state, it dose not need any regularization which was indispensable for the previous construction of closed string state. Our classical solution dose not suffer from the midpoint singularity and it obeys the equation of motion exactly.

It is proposed that, in CSFT or VSFT, on-shell closed string states are given by vertex operators at the midpoint on the identity string field [5, 22]. We have not yet understand the relation of our classical solution and its proposal.

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Appendix

A Derivation of Eqs. (2.26) and (3.59)

First, we find the following equation,

$$\frac{\sin x}{\cosh A - \cos x} = 2\sum_{n=1}^{\infty} e^{-nA}\sin(nx) \quad (A>0). \tag{A.67}$$

Integrating this equation, we obtain

$$\log(\cosh A - \cos x) = A - \log 2 - 2\sum_{n=1}^{\infty}\frac{e^{-nA}}{n}\cos(nx) \quad (A>0), \tag{A.68}$$

where the integral constant is given by comparing the values at $x = \pi/2$.

If $a > 0$, we can rewrite $\xi_a(\sigma)$ as

$$\begin{aligned}\xi_a(\sigma) &= \log(1+2a\cos^2\sigma) \\ &= \log a + \log\left[\left(1+\frac{1}{a}\right) - \cos(\pi - 2\sigma)\right].\end{aligned} \tag{A.69}$$

We apply Eq. (A.68) to the second term of Eq. (A.69) by taking $A = \log[(1+a+\sqrt{1+2a})/a]$

$$\begin{aligned}\xi_a(\sigma) &= \log\frac{1+a+\sqrt{1+2a}}{2} \\ &\quad -2\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\left(\frac{1+a-\sqrt{1+2a}}{a}\right)^n\cos(2n\sigma).\end{aligned} \tag{A.70}$$

If $0 > a > -1/2$, $\xi_a(\sigma)$ can be rewritten by

$$\xi_a(\sigma) = \log(-a) + \log\left[\left(-1-\frac{1}{a}\right) - \cos(2\sigma)\right]. \tag{A.71}$$

Applying Eq. (A.68) by taking $A = \log[(1+a+\sqrt{1+2a})/(-a)]$, we have the same expansion form of Eq. (A.70) as the case of $a > 0$.

Taking the limit $a \to 0$ in the right hand-side of Eq. (A.70), the result becomes zero and we find that Eq. (A.70) holds in the case of $a = 0$. Thus, we derive the formula of Eq. (3.59). We can obtain the formula of Eq. (2.26) by the derivative of Eq. (3.59) with σ.

B Derivation of Eqs. (2.27) and (2.28)

Using Eq. (2.26), we evaluate J_1 as follows

$$
\begin{aligned}
J_1 &= \int_0^{\frac{\pi}{2}} d\sigma \frac{2a\sin^2\sigma\cos\sigma}{1+2a\cos^2\sigma} \\
&= -\sum_{n=1}^{\infty}(-1)^n\left(\int_0^{\frac{\pi}{2}} d\sigma \sin(2n\sigma)\sin\sigma\right) Z(a)^n \\
&= \sum_{n=1}^{\infty}\left(\frac{1}{2n+1}+\frac{1}{2n-1}\right) Z(a)^n.
\end{aligned} \tag{B.72}
$$

Here, we have the following formulas,

$$
\begin{aligned}
&\sum_{n=0}^{\infty}\frac{1}{2n+1}A^{2n+1} = \frac{1}{2}\log\frac{1+A}{1-A} \qquad (0 \le A < 1), \\
&\sum_{n=0}^{\infty}(-1)^n\frac{1}{2n+1}A^{2n+1} = \arctan A \qquad (0 \le A < 1).
\end{aligned} \tag{B.73}
$$

Since $Z(a)$ takes the value of

$$
\begin{cases} 0 \le Z(a) < 1 & (a \ge 0) \\ -1 < Z(a) < 0 & (-1/2 < a < 0), \end{cases} \tag{B.74}
$$

we can derive Eq. (2.27) by the application of the above formulas to Eq. (B.72).

We consider the derivation of Eq. (2.28).

$$
\begin{aligned}
J_2 &= \int_0^{\frac{\pi}{2}} d\sigma \frac{2a\sin\sigma\cos\sigma}{1+2a\cos^2\sigma}\sin(3\sigma) \\
&= -\sum_{n=1}^{\infty}(-1)^n\left(\int_0^{\frac{\pi}{2}} d\sigma \sin(2n\sigma)\sin(3\sigma)\right) Z(a)^n \\
&= -\sum_{n=1}^{\infty}\left(\frac{1}{2n+3}+\frac{1}{2n-3}\right) Z(a)^n.
\end{aligned} \tag{B.75}
$$

Applying the formula of Eq. (B.73) to the above equation, we can obtain Eq. (2.28).

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